

Gravitational particle production in bouncing cosmologies

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Abstract

It is argued that the Universe reheating in bouncing cosmologies could be explained via gravitational particle production, as due to a sudden phase transition in the contracting regime. To this end, it is shown that, in the context of Loop Quantum Cosmology, gravitational production of massive particles conformally coupled with gravity in a matter-ekpyrotic bouncing Universe, where the sudden phase transition occurs in the contracting regime, yields a reheating temperature which is in good agreement with cosmological observations.

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1 Introduction

The issue of the Universe reheating in the matter bounce scenario (for descriptions of this scenario, see [1]), via gravitational particle production of light particles minimally coupled and nearly conformally coupled with gravity has been recently addressed, respectively in [2] and [3]. The idea is quite simple: to get efficient reheating one needs a non-adiabatic transition between two different phases, in order to obtain enough gravitational particle creation. This process is usually called *preheating*. At that time the Universe is far from thermal equilibrium, but then the created particles decay into very light particles which interact among themselves, thus producing a relativistic plasma in thermal equilibrium and evolving like radiation. Finally, when the energy density of this relativistic plasma starts to dominate that of the background, the Universe is reheated and enters in a radiation dominated phase which matches well with the corresponding one for the hot Friedmann Universe.

In the matter bounce scenario the non-adiabatic transition could be produced in the contracting phase. In fact, a transition from matter-domination (this phase is essential in order to guarantee scale invariance of the power spectrum of perturbations, because modes that leave the Hubble radius during this phase have a flat power spectrum [4]), to an ekpyrotic phase with equation of state $P = \omega\rho$ where $\omega > 1$ could be assumed in the contracting regime. The model thus obtained is being called in the literature a matter-ekpyrotic bounce scenario [5], and since in the ekpyrotic phase the energy density of the field, namely φ , evolves like $\rho_\varphi \sim a^{-3(1+\omega)}$, which in the contracting phase increases faster than a^{-6} , anisotropies do become negligible. (Note that the energy density of the anisotropies grows in the contracting phase as a^{-6} , that is, faster than the matter energy density, and thus, without an ekpyrotic transition the isotropy of the bounce would be destroyed; this is the so-called Belinsky-Khalatnikov-Lifshitz instability [6].) Moreover, the field energy density also grows faster than that of the particles produced, what means that the field dominates the Universe evolution in the contracting phase. But when the Universe bounces, the energy density of the relativistic plasma generated by the decay of the particles created due to the phase transition will eventually dominate, and thus, the Universe will finally become radiation-dominated, in a way that may perfectly match the results for the hot Friedmann Universe.

This situation is similar to the one occurring in inflationary cosmology. To wit, for inflationary models with potentials not having a minimum (the so-called non-oscillatory models [7]), there is an abrupt transition from a quasi de Sitter regime to a kinetic-dominated period. During this transition massless, nearly conformally coupled particles are created which energy density evolves like $\rho_r \sim a^{-4}$. On the other hand, when the inflaton field, Φ , enters a kinetic-dominated period its energy density behaves like $\rho_\Phi \sim a^{-6}$ [8, 9, 10], what means that the inflation energy density decreases faster than that of the radiated particles, and thus, the Universe will become radiation dominated, to eventually match the hot Friedmann Universe. On the other hand, in inflationary models with potentials which do exhibit a minimum, adiabaticity is broken when the inflaton field oscillates [11]; at that moment massive particles are created and, after the preheating stage, they decay into light particles which thermalize and whose energy density eventually dominates the one of the inflaton. Therefore, the Universe becomes radiation dominated, too.

In the present study we will consider, in the flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, the matter-ekpyrotic bounce scenario in the context of Loop Quantum Cosmology [12], where a phase transition to the ekpyrotic phase is produced in the contracting regime. One of the most remarkable properties of this model is its simplicity, in the sense that, in the non-singular bounce the adiabaticity is not broken because it is well-known that given the Equation of State $P = w\rho$, dealing with Loop Quantum Cosmology, the scale factor evolves as [13] $a(t) = \left(\frac{3\rho_c}{4M_{Pl}^2} (1+w)^2 t^2 + 1 \right)^{\frac{1}{3(1+w)}}$, where ρ_c is the energy density at bouncing time and M_{Pl} is the Planck mass. One can easily see that at bouncing time ($t = 0$) the scale factor is an smooth function what preserves the adiabaticity. This situation does not happen, i.e., the transition from contraction to expansion is not adiabatic, in other bouncing cosmologies, for example, in cosmologies within the framework of General Relativity where the violation of the null energy condition at the bouncing time (needed in order to have a bounce in the flat FLRW geometry), could be possible incorporating new forms of matter such as phantom [14] or quintom fields [15], Galileons [16] or phantom condensates [17].

In our simple study, we also deal with heavy massive particles conformally coupled with gravity, because in Fourier space, these particles can be depicted as a set of harmonic oscillators whose posi-

tive frequency is time dependent. This allows to easily interpret the number of particles created and its energy density as the 00 component of the stress-energy tensor, where the vacuum zero point energy of the set of oscillators must be subtracted, in order to get a well-defined quantity (see, e.g., [18]). Then, the energy density of the created particles will be calculated and, in the expanding phase, when it is of the same order as the background energy, the Universe will get reheated, with a temperature of the same order of this energy density power $1/4$. We will see that the reheating temperature will basically depend on the energy density of the background when the sudden phase transition is produced, and thus, by properly choosing this energy density, the reheating temperature will be shown to match well that of astronomical observations.

Units used in the work are: $\hbar = c = 1$.

2 Particle production of massive conformally coupled particles

Let $\phi \equiv \frac{\chi}{a}$ be a massive scalar field conformally coupled with gravity, a being the scale factor. In Fourier space, the corresponding Klein-Gordon equation, in the FLRW spacetime, is given by the following set of harmonic oscillators, with time dependent frequency [19]:

$$\chi_k'' + \omega_k^2(\eta)\chi_k = 0, \quad (1)$$

where $\omega_k^2(\eta) = |k|^2 + m^2 a^2(\eta)$, being η the conformal time and m the mass of the field.

Using the instantaneous diagonalization method [20], the square of the β -Bogoliubov coefficient is given by

$$|\beta_k(\eta)|^2 = \frac{1}{\omega_k(\eta)} \left[\frac{1}{2} (|\chi_k'(\eta)|^2 + \omega_k^2(\eta)|\chi_k(\eta)|^2) - \frac{1}{2}\omega_k(\eta) \right]. \quad (2)$$

We assume that at some given time, η_E , there is a sudden phase transition between two adiabatic phases. Note that, as $\frac{d^s \omega_k}{d\eta^s} \sim H^s m$ (H being the Hubble parameter), if one assumes $H \leq H_{max} \ll m$ (i.e., the generated particles have a large mass), where H_{max} is the maximum value of the Hubble parameter—what always happens in holonomy corrected Loop Quantum Cosmology because in the plane (H, ρ) the modified Friedmann equation depicts an ellipse that is a bounded curve [21]—then, for $n = -1 + (r + 1)\alpha + (s + 1)\beta$ with $\alpha + \beta - 1 > 0$, one gets

$$\frac{d^n \omega_k}{d\eta^n} \ll \left(\frac{d^r \omega_k}{d\eta^r} \right)^\alpha \left(\frac{d^s \omega_k}{d\eta^s} \right)^\beta, \quad (3)$$

in particular, for $n = 1$, $\alpha = 2$ and $r = s = \beta = 0$, one has $\omega_k' \ll \omega_k^2$.

During the adiabatic regimes we use the first order WBK solution [22] of (1)

$$\chi_{1,k}(\eta) = \frac{1}{\sqrt{2W_{1,k}(\eta)}} e^{-i \int^\eta W_{1,k}(\eta) d\eta} \text{ where } W_{1,k} = \omega_k - \frac{1}{2\omega_k} \left[\frac{\omega_k''}{2\omega_k} - \frac{3}{4} \left(\frac{\omega_k'}{\omega_k} \right)^2 \right], \quad (4)$$

together with its conjugate, to approximate the mode solutions.

In the first phase, the mode corresponding to the vacuum state is approximated by $\chi_{1,k}$. Before the sudden phase transition this mode becomes $a_k \chi_{1,k} + b_k \chi_{1,k}^*$, with

$$b_k = -i\mathcal{W}[\chi_{1,k}(\eta_E^-); \chi_{1,k}(\eta_E^+)], \quad (5)$$

where \mathcal{W} is the Wronskian and $\chi_{1,k}(\eta_E^-)$ (resp. $\chi_{1,k}(\eta_E^+)$) is the value of the mode just before (after) the time η_E . A simple calculation using the relation $|a_k|^2 - |b_k|^2 = 1$, yields

$$|\beta_k(\eta)|^2 = \frac{1}{2\omega_k(\eta)} [|\chi'_{1,k}(\eta)|^2 + \omega_k^2(\eta)|\chi_{1,k}(\eta)|^2] - \frac{1}{2} + \frac{|b_k|^2}{\omega_k(\eta)} [|\chi'_{1,k}(\eta)|^2 + \omega_k^2(\eta)|\chi_{1,k}(\eta)|^2] + \frac{1}{\omega_k(\eta)} \mathcal{R}e(a_k b_k^* [(\chi'_{1,k}(\eta))^2 + \omega_k^2(\eta)(\chi_{1,k}(\eta))^2]) = |b_k|^2 + \mathcal{O}\left(\frac{\omega'_k(\eta)}{\omega_k^2(\eta)}\right) \cong |b_k|^2. \quad (6)$$

Assuming that the derivative of a has a discontinuity at some time value, $\eta = \eta_E$ (namely that the Hubble parameter is discontinuous at $\eta = \eta_E$), one gets

$$|\beta_k(\eta)|^2 \cong \frac{(\omega'_k(\eta_E^+) - \omega'_k(\eta_E^-))^2}{16\omega_k^4(\eta_E)}, \quad (7)$$

which, in terms of the cosmic time, reads

$$|\beta_k(t)|^2 \cong \frac{m^4 a_E^6 (H_E^+ - H_E^-)^2}{16(|k|^2 + m^2 a_E^2)^3}, \quad (8)$$

where $a_E \equiv a(t_E)$ and $H_E^\pm \equiv H(t_E^\pm)$.

When the Hubble parameter is continuous but its derivative is discontinuous, that is, the scalar curvature is discontinuous, at some instant t_E , a similar calculation leads to

$$|\beta_k(t)|^2 \cong \frac{m^4 a_E^8 (R_E^+ - R_E^-)^2}{2304(|k|^2 + m^2 a_E^2)^4} = \frac{m^4 a_E^8 (\dot{H}_E^+ - \dot{H}_E^-)^2}{64(|k|^2 + m^2 a_E^2)^4}, \quad (9)$$

where $R(t)$ is the scalar curvature and $R_E^\pm \equiv R(t_E^\pm)$.

On the other hand, the density of produced particles and their corresponding energy density are respectively given by [23]

$$N_\chi(\eta) = \frac{1}{(2\pi a(\eta))^3} \int_{\mathbb{R}^3} |\beta_k(\eta)|^2 d^3k; \quad \rho_\chi(\eta) = \frac{1}{(2\pi a(\eta))^3 a(\eta)} \int_{\mathbb{R}^3} \omega_k(\eta) |\beta_k(\eta)|^2 d^3k. \quad (10)$$

Note that, from the definition of the β -Bogoliubov coefficients (2), we can see that the energy density $\rho(t)$ is the 00 component of the stress-energy tensor where, in order to obtain a convergent quantity, one needs to subtract the vacuum zero-point energy [18].

As a consequence, in the case when the Hubble parameter has a discontinuity at $t = t_E$, from formula (8) one obtains, in cosmic time,

$$N_\chi(t) = \frac{m(H_E^+ - H_E^-)^2}{512\pi} \left(\frac{a_E}{a(t)}\right)^3; \quad \rho_\chi(t) = \frac{m^2(H_E^+ - H_E^-)^2}{32\pi^2} \left(\frac{a_E}{a(t)}\right)^4 I_3\left(\frac{a(t)}{a_E}\right), \quad (11)$$

where

$$I_3\left(\frac{a(t)}{a_E}\right) \equiv \int_0^\infty \frac{x^2 \sqrt{x^2 + \left(\frac{a(t)}{a_E}\right)^2}}{(x^2 + 1)^3} dx = \begin{cases} \frac{\left(\frac{a(t)}{a_E}\right)^4 \arctan\left(\sqrt{\left(\frac{a(t)}{a_E}\right)^2 - 1}\right)}{8\left[\left(\frac{a(t)}{a_E}\right)^2 - 1\right]\sqrt{\left(\frac{a(t)}{a_E}\right)^2 - 1}} + \frac{\left(\frac{a(t)}{a_E}\right)^2 - 2}{8\left[\left(\frac{a(t)}{a_E}\right)^2 - 1\right]}, & \text{for } \frac{a(t)}{a_E} > 1, \\ 1/3, & \text{for } \frac{a(t)}{a_E} = 1, \\ \frac{\left(\frac{a(t)}{a_E}\right)^4 \tanh^{-1}\left(\sqrt{1 - \left(\frac{a(t)}{a_E}\right)^2}\right)}{8\left[\left(\frac{a(t)}{a_E}\right)^2 - 1\right]\sqrt{1 - \left(\frac{a(t)}{a_E}\right)^2}} + \frac{\left(\frac{a(t)}{a_E}\right)^2 - 2}{8\left[\left(\frac{a(t)}{a_E}\right)^2 - 1\right]}, & \text{for } \frac{a(t)}{a_E} < 1. \end{cases} \quad (12)$$

From this result it follows that, when $\frac{a(t)}{a_E} \ll 1$,

$$\rho_\chi(t) \cong \frac{m^2(H_E^+ - H_E^-)^2}{128\pi^2} \left(\frac{a(t)}{a_E} \right)^4$$

and, consequently, if the sudden phase transition takes place in the contracting phase, then the generated particles will evolve, in a part of the contracting and expanding phase, exactly like radiation does. However, if these particles do not decay as a relativistic plasma then, when $\frac{a(t)}{a_E} \gg 1$, they will evolve as matter since, in this case

$$\rho_\chi(t) \cong mN_\chi(t) = \frac{m(H_E^+ - H_E^-)^2}{512\pi} \left(\frac{a_E}{a(t)} \right)^3.$$

The same happens when the Hubble parameter is continuous but the scalar curvature has a discontinuity at $t = t_E$. In such case, from formula (9), one obtains, in cosmic time,

$$N_\chi(t) = \frac{(\dot{H}_E^+ - \dot{H}_E^-)^2}{4096m\pi} \left(\frac{a_E}{a(t)} \right)^3; \quad \rho_\chi(t) = \frac{(\dot{H}_E^+ - \dot{H}_E^-)^2}{128\pi^2} \left(\frac{a_E}{a(t)} \right)^4 I_4 \left(\frac{a(t)}{a_E} \right), \quad (13)$$

where $I_4 \left(\frac{a(t)}{a_E} \right) \equiv \int_0^\infty \frac{x^2 \sqrt{x^2 + \left(\frac{a(t)}{a_E} \right)^2}}{(x^2 + 1)^4} dx$, is related with $I_3 \left(\frac{a(t)}{a_E} \right)$ through the formula

$$I_4(y) = \frac{1}{6y} \frac{d(y^2 I_3(y))}{dy}.$$

In the asymptotic case $\frac{a(t)}{a_E} \ll 1$, one has

$$\rho_\chi(t) \cong \frac{(\dot{H}_E^+ - \dot{H}_E^-)^2}{1536\pi^2} \left(\frac{a_E}{a(t)} \right)^4$$

and, when $\frac{a(t)}{a_E} \gg 1$,

$$\rho_\chi(t) \cong mN_\chi(t) = \frac{(\dot{H}_E^+ - \dot{H}_E^-)^2}{4096\pi} \left(\frac{a_E}{a(t)} \right)^3.$$

Finally, it is important to stress that, when the Hubble parameter is continuous but the curvature is discontinuous, the energy density does not depend on the mass of the field.

3 Bouncing models

In this Section we will here study two simple bouncing models coming from Loop Quantum Cosmology, where holonomy corrections modify Friedmann's equation as follows [24]

$$H^2 = \frac{\rho}{3M_{Pl}^2} \left(1 - \frac{\rho}{\rho_c} \right), \quad (14)$$

being M_{Pl} the reduced Planck mass and ρ_c the so-called *critical energy density*, the value of the energy density at the bouncing time, and its maximum value. From this equation, it is clear that when $\rho \ll \rho_c$ one recovers the standard Friedmann equation, that is, holonomy corrections can then be disregarded. Moreover, we can see that equation (14) depicts an ellipse in the plane (H, ρ) [21].

As we have explained in the Introduction, holonomy corrected Loop Quantum Cosmology provides non-singular bounce that preserves adiabaticity. Effectively, if during the transition from contraction to expansion, the Universe is dominated by a barotropic fluid with Equation of State $P = w\rho$, the solution of (14) will be given by [13]

$$a(t) = \left(\frac{3\rho_c}{4M_{Pl}^2} (1+w)^2 t^2 + 1 \right)^{\frac{1}{3(1+w)}}, \quad (15)$$

which has an smooth behavior at the bouncing time $t = 0$, and thus, leads to an adiabatic transition.

In both models we will assume that, in the contracting phase, there is a sudden transition from the matter domination phase to the ekpyrotic one, that breakdowns the adiabaticity. And since, as we have already seen, holonomy corrected Loop Quantum Cosmology preserves adiabaticity at the bouncing time, in both models all the particles are created, in the contracting regime, during this sudden transition.

3.1 First model

In this model we will consider a discontinuous Hubble parameter at $t = t_E$ which scale factor is given by

$$a(t) = \begin{cases} a_E \left(\frac{t}{t_E} \right)^{2/3}, & \text{when } t < t_E, \\ \left(\frac{3\rho_c}{4M_{Pl}^2} (1+w)^2 t^2 + 1 \right)^{\frac{1}{3(1+w)}}, & \text{when } t \geq t_E, \end{cases} \quad (16)$$

where we assume $w > 1$ in order to have an ekpyrotic phase.

Note that for $t < t_E$, the scale factor is the solution of the standard Friedmann equation $H^2 = \frac{\rho}{3M_{Pl}^2}$, for a matter dominated Universe ($P = 0$), and for $t > t_E$, the scale factor is the solution of the holonomy corrected Friedmann equation (14), for an ekpyrotic Universe whose Equation of State is $P = w\rho$.

Assuming moreover that the sudden transition is produced at very early times, $\frac{3\rho_c}{4M_{Pl}^2} (1+w)^2 t^2 \gg 1$, one obtains $H_E^+ - H_E^- = -\frac{w}{1+w} H_E^-$, and thus, from equation (11), the density of generated particles is

$$N_\chi(t) = \frac{mw^2(H_E^-)^2}{512\pi(1+w)^2} \left(\frac{a_E}{a(t)} \right)^3. \quad (17)$$

Now we will show that this amount of produced particles is enough to reheat our Universe. To prove our statement, first of all, note that immediately after the transition, the energy density of the produced particles and the background energy density are, respectively,

$$\rho_\chi(t_E^+) \cong \frac{m^2 w^2 \rho_E^-}{96\pi M_{Pl}^2 (1+w)^2}, \quad \text{and} \quad \rho(t_E^+) = \frac{\rho_E^-}{(1+w)^2}, \quad (18)$$

where ρ_E^- is the background energy density at the end of the matter domination phase.

Then, in order to have a sub-dominant energy density for the particles created at the beginning of the ekpyrotic phase—so that back-reaction effects can be disregarded—one has to assume that $M_{Pl} \gg mw$. Since in the contracting phase the background energy density increases as $\left(\frac{a_E}{a(t)}\right)^{3(1+w)}$, it will be dominant during this phase, and only in the expanding one will both energy densities be of the same order, at some time $t = t_R$. But then, the order of the reheating temperature may be obtained as $T_R \sim \rho_\chi^{1/4}(t_R)$.

Secondly, as we have already explained in the Introduction, the particles created are far from being in thermal equilibrium. The more massive particles will decay into lighter particles which will interact through multiple scattering, thus redistributing their energies until they yield a relativistic plasma in thermal equilibrium (see [25] for a very detailed description of this process).

Let Γ be the decay rate of massive χ -particles. The decay will be accomplished at time t_{dec} when $\Gamma|t_{dec} - t_E| \cong 1$. If, in order to simplify things (even though it is not essential), we assume this happens in the contracting phase when holonomy corrections can be neglected, then it turns out that, as for $t > t_E$ one has $H \cong \frac{2}{3(1+w)t}$, one gets

$$\frac{2}{3(1+w)} \frac{H_E^+ - H_{dec}}{H_E^+ H_{dec}} \cong \Gamma^{-1} \implies H_{dec} \cong \frac{2\Gamma H_E^+}{2\Gamma + 3(1+w)H_E^+}. \quad (19)$$

This means that the decay rate must satisfy the constrain

$$0 < |H_{dec}| \ll \sqrt{\frac{\rho_c}{M_{Pl}^2}} \implies \Gamma \gg -\frac{3}{2}(1+w) \frac{H_E^+}{\sqrt{\rho_c} + M_{Pl}H_E^+}, \quad (20)$$

because we are assuming that the decay is produced, in the contracting phase, whenever holonomy corrections can be disregarded.

Then, when the decay is effective, the background energy density reads

$$\rho_{dec} \cong 3M_{pl}^2 \left(\frac{2\Gamma H_E^+}{2\Gamma + 3(1+w)H_E^+} \right)^2 \ll \rho_c. \quad (21)$$

And thus, once the decay has taken place, a re-distribution of energies among the different particles occurs—this is the *kinetic equilibrium*—and, also, an increase in the number of particles—the *chemical equilibrium*. That is, in such process, in order to obtain a relativistic plasma in thermal equilibrium, both number-conserving and number-violating reactions are definitely involved.

Finally, to calculate the reheating temperature, one has to impose that both the energy densities of the relativistic plasma and of the background are of the same order (this will happen, in the expanding phase, when $a(t_R) \gg a_E$), namely

$$\rho(t_R) = \frac{\rho_E^-}{(1+w)^2} \left(\frac{a_E}{a(t_R)} \right)^{3(1+w)} \sim \rho_\chi(t_R) \cong \frac{m^2 w^2 \rho_E^-}{M_{Pl}^2 (1+w)^2} \left(\frac{a_E}{a(t_R)} \right)^4, \quad (22)$$

what means that

$$\frac{a_E}{a(t_R)} \sim \left(\frac{m^2 w^2}{M_{Pl}^2} \right)^{\frac{1}{3w-1}}. \quad (23)$$

And thus, the reheating temperature will become

$$T_R \sim \rho_\chi^{1/4}(t_R) \sim \left[\frac{\rho_E^-}{M_{Pl}^4(1+w)^2} \left(\frac{m^2 w^2}{M_{Pl}^2} \right)^{\frac{3(w+1)}{3w-1}} \right]^{1/4} M_{Pl}. \quad (24)$$

Note that in the asymptotic case $w \gg 1$, the reheating temperature has this simple expression

$$T_R \sim \left(\frac{m^2 \rho_E^-}{M_{Pl}^6} \right)^{1/4} M_{Pl}, \quad (25)$$

and in this case, assuming from recent observations that $T_R \sim 10^{-7} M_{Pl}$ (see [2]), one concludes that the energy density of the background at the end of the matter domination stage has to be necessarily of the order

$$\rho_E^- \sim \left(10^{-28} \frac{M_{Pl}^2}{m^2} \right) M_{Pl}^4,$$

that is, our model will match with observation, i.e., the amount of created particles is enough to reheat the Universe, when the phase transition is produced at very low energy densities, or equivalently, at very early times in the contracting phase.

To end this example it is instructive to compare it with its inflationary dual, that is, a Universe depicted by a sudden transition from the de Sitter phase to a radiation-dominated one. Suppose inflation is produced by an false vacuum energy density ρ_E , then the scale factor is given by [8]

$$a(\eta) = \begin{cases} \frac{1}{\eta H_E} & \text{for } \eta < \eta_E < 0 \\ H_E(\eta - \eta_E) + \frac{1}{\eta_E H_E} & \text{for } \eta > \eta_E, \end{cases} \quad (26)$$

where H_E is the value of the Hubble parameter during the de Sitter phase. Then, for heavy massive ($m \gg H_E$) particles conformally coupled with gravity, since $H(\eta)$ is discontinuous at $\eta = \eta_E$, one could use formula (11) to obtain $\rho_\chi(t_E) \sim H_E^2 m^2$, and thus, after decaying into light particles and thermalize the reheating temperature is, as in matter-ekpyrotic bounce scenario with $w \gg 1$, given by

$$T_R \sim (H_E m)^{1/2} = \left(\frac{m^2 \rho_E}{M_{Pl}^6} \right)^{1/4} M_{Pl}, \quad (27)$$

which is another manifestation of the duality between the de Sitter regime in the expanding phase and the matter-domination in the contracting one, pointed out, for the first time, in [26].

3.2 Second model

As second model of matter-ekpyrotic bouncing scenario in Loop Quantum Cosmology we will consider a sudden transition where the Hubble parameter is continuous but the scalar curvature has a discontinuity at $t = t_E$. If one considers equation (14) with and Equation of State of the form

$$P(\rho) = \begin{cases} 0 & \text{when } \rho < \rho_E \\ w\rho & \text{when } \rho > \rho_E, \end{cases} \quad (28)$$

in the contracting phase, and $P(\rho) = w\rho$ in the expanding one, the scale factor will be given by

$$a(t) = \begin{cases} a_E \left(\frac{t-t_0}{t_E-t_0} \right)^{2/3}, & \text{when } t < t_E, \\ \left(\frac{3\rho_c}{4M_{Pl}^2} (1+w)^2 t^2 + 1 \right)^{\frac{1}{3(1+w)}}, & \text{when } t \geq t_E, \end{cases} \quad (29)$$

where $t_0 \equiv t_E - \frac{2}{3H_E}$, being $H_E = \sqrt{\frac{\rho_E}{3M_{Pl}^2}}$ the value of the Hubble parameter at the time of the phase transition.

Note that, since we are assuming that the transition is produced at very early times, during the matter domination, holonomy corrections could safely be disregarded, and thus, the exact value of the scale factor $a(t)$ is approximately given by $a_E \left(\frac{t-t_0}{t_E-t_0} \right)^{2/3}$.

In this case, $R_E^+ - R_E^- = -9wH_E^2$, and then the density of produced particles is

$$N_\chi(t) = \frac{81w^2H_E^4}{4096m\pi} \left(\frac{a_E}{a(t)} \right)^3. \quad (30)$$

On the other hand, at the transition time the energy density of these generated particles evolves as

$$\rho_\chi(t) \sim \frac{w^2\rho_E^2}{M_{Pl}^4}. \quad (31)$$

As a consequence, such energy density will be sub-dominant at the beginning of the ekpyrotic phase, provided $w^2\rho_E \ll M_{Pl}^4$. And since the background energy density increases fastly in the contracting regime and also decreases fastly in the expanding one, the Universe will eventually reheat, in the expanding phase, when both these energy densities are of the same order. This will happen when $a(t_R) \gg a_E$.

Imposing this condition, one obtains

$$\frac{a_E}{a(t_R)} \sim \left(\frac{w^2\rho_E}{M_{Pl}^4} \right)^{\frac{1}{3w-1}}, \quad (32)$$

and thus, the reheating temperature will be

$$T_R \sim \rho_\chi^{1/4}(t_R) \sim \left[\frac{\rho_E}{M_{Pl}^4} \left(\frac{\rho_E w^2}{M_{Pl}^4} \right)^{\frac{3(w+1)}{3w-1}} \right]^{1/4} M_{Pl}. \quad (33)$$

Finally, note that in the asymptotic case, $w \gg 1$, the reheating temperature acquires the simple form

$$T_R \sim \left(\frac{w\rho_E}{M_{Pl}^4} \right)^{1/2} M_{Pl} \quad (34)$$

and, in this case, assuming $T_R \sim 10^{-7}M_{Pl}$, it turns out that, in order to match with recent observational data, the energy density of the background at the phase transition must be of the order

$$\rho_E \sim 10^{-14} \frac{1}{w} M_{Pl}^4.$$

4 Some remarks about massless particle production

When one deals with particles non-conformally coupled with gravity, the diagonalization method results, in general, in a divergent number of produced particles. This means that, in such case, the concept of created particles has to be defined in a different way. We believe that, in the case of massless particles non-conformally coupled with gravity, the modes associated to the vacuum must be those which, at an early time, satisfy the asymptotic condition $\chi_k(\eta) \rightarrow \frac{e^{-i|k|\eta}}{\sqrt{2|k|}}$.

Since the Klein-Gordon equation for massless particles reads

$$\chi_k'' + \left(|k|^2 - (1 - 6\xi) \frac{a''}{a} \right) \chi_k = 0, \quad (35)$$

where ξ is the coupling constant (equal to $1/6$ when the coupling is conformal), with our definition, the modes that define the vacuum state in the matter dominated phase are

$$\chi_k^{matt}(\eta) = e^{-i(\frac{\pi\nu_{matt}}{2} + \frac{\pi}{4})} \sqrt{\frac{\pi\eta}{4}} H_{\nu_{matt}}^{(2)}(|k|\eta), \quad (36)$$

where $H_{\nu_{matt}}^{(2)}$ is the Hankel's function [27], with $\nu_{matt} \equiv \sqrt{\frac{9}{4} - 12\xi}$. Here, we will assume that $\xi < \frac{3}{16}$, in order that ν_{matt} becomes a real number. Note that these modes are the same that define the vacuum in the de Sitter regime in the expanding phase because, in both cases, $\frac{a''}{a} = \frac{2}{\eta}$, this is the duality between matter-domination during contraction and de Sitter phase during expansion [26].

On the other hand, if the phase transition takes place at very early times, when holonomy corrections can be safely disregarded, then the modes which define the vacuum state in the ekpyrotic regime, just following the phase transition, are given by

$$\chi_k^{ekpy}(\eta) = e^{-i(\frac{\pi\nu_{ekpy}}{2} + \frac{\pi}{4})} \sqrt{\frac{\pi\eta}{4}} H_{\nu_{ekpy}}^{(2)}(|k|\eta), \quad (37)$$

with $\nu_{ekpy} \equiv \sqrt{\frac{1}{4} + \frac{2(1-3w)}{(1+3w)^2}(1-6\xi)}$. Consequently, the squared modulus of the beta Bogoliubov coefficients are calculated as

$$|\beta_k|^2 = \left| \mathcal{W} \left[\sqrt{\frac{\pi\eta_E}{4}} H_{\nu_{matt}}^{(2)}(|k|\eta_E); \sqrt{\frac{\pi\eta_E}{4}} H_{\nu_{ekpy}}^{(2)}(|k|\eta_E) \right] \right|^2. \quad (38)$$

What is remarkable is that for $w \gg 1$ one gets $\nu_{ekpy} \cong \frac{1}{2}$, and thus, $\chi_k^{ekpy}(\eta) \cong \frac{e^{-i|k|\eta}}{\sqrt{2|k|}}$. That is, when $w \gg 1$ the phase transition from the matter to the ekpyrotic phase in the contracting regime is exactly the same as the phase transition from the de Sitter to a radiation dominated phase in the expanding one. This last phase transition has been studied in several works, to explain the reheating in inflationary models via gravitational particle production (see, e.g., [8, 28, 29, 19]).

Then, for $w \gg 1$, when one consider modes well outside of the Hubble radius, i.e., that satisfy $|k\eta_E| \ll 1$, we can use the formula for small arguments [27]

$$H_\nu^{(2)}(z) \cong \frac{i}{\pi} (z/2)^{-\nu} \Gamma(\nu) - \frac{ie^{i\pi\nu}}{\sin(\pi\nu)} (z/2)^\nu \frac{1}{\Gamma(\nu+1)}, \quad (39)$$

to obtain, after matching at η_E with the mode $\alpha_k \frac{e^{-i|k|\eta}}{\sqrt{2|k|}} + \beta_k \frac{e^{i|k|\eta}}{\sqrt{2|k|}}$, or using Eq. (38), the formula

$$|\beta_k|^2 \cong \frac{1}{16\pi} (|k\eta_E|/2)^{-2\nu_{matt}-1} \Gamma^2(\nu_{matt}) (1/2 - \nu_{matt})^2. \quad (40)$$

And in the opposite case, $|k\eta_E| \gg 1$ —i.e., for models well inside the Hubble radius—from the asymptotic expression for large arguments [27]

$$H_\nu^{(2)}(z) \cong e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} \sqrt{\frac{2}{\pi z}} \left(1 - i \frac{4\nu^2 - 1}{8z}\right) e^{-iz}, \quad (41)$$

after matching at η_E , one obtains

$$|\beta_k|^2 \cong \frac{(4\nu_{matt}^2 - 1)^2}{4|k|^4 \eta_E^4}. \quad (42)$$

It follows from these formulas that the energy density of the produced particles is always ultraviolet divergent, and also, that for $\nu_{matt} \geq 1$ (i.e., $\xi \leq \frac{5}{48}$) the number of created particles is infrared divergent, although their energy density is only infrared divergent for $\nu_{matt} \geq 3/2$, i.e., for $\xi \leq 0$, what includes massless particles minimally coupled with gravity.

To avoid infrared divergencies one must assume that the coupling constant ξ belongs in the range $(5/48, 3/16)$ and, in order to have a finite energy density one needs to assume, as was shown in [8], that the scalar curvature is regular (continuous, at the very least) during the phase transition.

Simple formulas can be obtained if one only considers the production of particles whose modes leave the Hubble radius before the phase transition, i.e., for modes satisfying $|k\eta_E| < 1$. In this case, when $5/48 < \xi < 3/16 \iff 0 < \nu_{matt} < 1$ (to avoid infrared divergencies), using Eq. (40) one gets

$$N_\chi(t) \cong \frac{4^{\nu_{matt}}}{256\pi^3(1 - \nu_{matt})} \Gamma^2(\nu_{matt}) \left(\frac{1}{2} - \nu_{matt}\right)^2 |H_E|^3 \left(\frac{a_E}{a(t)}\right)^3, \quad (43)$$

and

$$\rho_\chi(t) \cong \frac{4^{\nu_{matt}}}{256\pi^3(3 - 2\nu_{matt})} \Gamma^2(\nu_{matt}) \left(\frac{1}{2} - \nu_{matt}\right)^2 H_E^4 \left(\frac{a_E}{a(t)}\right)^4. \quad (44)$$

Note that the case $\nu_{matt} = 1/2 \iff \xi = \frac{1}{6}$ is the conformally coupled one, and thus, there is not particle production.

Finally, provided the phase transition is smooth enough, the energy density of the massless particles produced will be of the order $\rho_\chi(t) \sim \left(\frac{1}{2} - \nu_{matt}\right)^2 \frac{\rho_E^2}{M_{Pl}^4} \left(\frac{a_E}{a(t)}\right)^4 \sim \left(\frac{1}{2} - \nu_{matt}\right)^2 \frac{\rho_E^2}{M_{Pl}^4}$, because for $w \gg 1$ the approximation $\left(\frac{a_E}{a(t)}\right)^4 \cong 1$ holds in the ekpyrotic case. On its turn, the reheating temperature will be of the order

$$T_R \sim \left(\left|\frac{1}{2} - \nu_{matt}\right| \frac{\rho_E}{M_{Pl}^4}\right)^{1/2} M_{Pl}, \quad (45)$$

which coincides, with the reheating temperature obtained in a transition from the de Sitter phase to a radiation-dominated one, because in this inflationary case one has the well-known result [9, 19, 28]

$\rho_\chi \sim \left(\frac{1}{2} - \nu\right)^2 H_E^4$ (compare with Eq. (44)), with $\nu = \nu_{\text{matt}} = \sqrt{\frac{9}{4} - 12\xi}$. What shows once again the duality between the transition from the de Sitter regime to a radiation-dominated Universe in the expanding phase, and the transition from matter domination to an ekpyrotic regime in the contracting phase.

5 Conclusions

We have shown in this paper that, for models arising in holonomy corrected Loop Quantum Cosmology, a simple phase transition which takes place in the contracting phase, from matter domination to an ekpyrotic regime, can lead in fact to particle production of heavy particles conformally coupled with gravity in sufficient amount (see formulae (17) and (30)) and with enough energy density to reheat the Universe in the expanding phase. This is quite a remarkable conclusion. Moreover, as a bonus we see that the expression of the reheating temperature is actually quite simple (see formulae (25), (34) and (45)), what does not happen in other bouncing models where adiabaticity is broken at bouncing time (see for instance, formula (60) of [2], where production of minimally coupled particles and reheating are studied in the two-fields matter bounce scenario). In our results, the reheating temperature basically depends on the energy density scale at which the phase transition occurs, what leaves us with just one degree of freedom in order to match this theoretical value with the observational data, what makes of the model a reasonably predictive one. Finally, we have emphasized that the reheating temperature obtained in our model is the same obtained in inflationary cosmology via gravitational particle production, what shows the dual relation that exists between the matter-ekpyrotic bouncing scenario in Loop Quantum Cosmology and the so-called non-oscillatory inflationary models.

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